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Level 2 Certificate in Further Mathematics

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Paper 2 8360/2

Mark Scheme

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Glossary for Mark Schemes

These examinations are marked in such a way as to award positive achievement wherever possible. Thus, for these papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
M Dep	A method mark dependent on a previous method mark being awarded.
B Dep	A mark that can only be awarded if a previous independent mark has been awarded.
ft	Follow through marks. Marks awarded following a mistake in an earlier step.
SC	Special case. Marks awarded within the scheme for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg, accept 0.5 as well as $\frac{1}{2}$

Paper 2 - Calculator

Q	Answer	Mark	Comments
1	Radius = $\sqrt{36}$ **or** 6	B1	Diameter = $2\sqrt{36}$ **or** 12
	2 (×) π (×) their radius	M1	π (×) their diameter
	12π **or** [37.68, 37.704]	A1	
2	$15x^2 - 8x$	B2	B1 Only one term correct
3	8^2 **or** 4^2 **or** 64 **or** 16 **or** 80 **or** (-8^2) **or** (-4^2)	M1	
	$\sqrt{\text{their } 8^2 + \text{their } 4^2}$	M1 Dep	
	8.944(…) **or** $\sqrt{80}$	A1	oe eg $4\sqrt{5}$ This mark is implied by 8.94
	8.94	B1 ft	ft From any value > 3sf seen or any value given as a surd that is rounded to 3sf
4(a)	Positive	B1	Do not allow if more than one answer selected
4(b)	Negative	B1	Do not allow if more than one answer selected
4(c)	One positive and one negative	B1	Do not allow if more than one answer selected
4(d)	0	B1	Do not allow if more than one answer selected
4(e)	$y = -3$	B1	Do not allow if more than one answer selected

Q	Answer	Mark	Comments
5(a)	Angle $ACP = x$ or angle PAC (base angles of) isosceles triangle (are equal)	M1	
	Angle $APC = 180 - 2x$ angle sum of triangle (= 180°) **and** angle $BPC = 2x$ angles on straight line (add to 180°)	M1 Dep	$BPC = 2x$ external angle of triangle (= sum of interior opposite angles)
	Angle $ABC = 2x$ or angle BPC (base angles of) isosceles triangle (are equal)	A1	SC2 'Correct' response but has reason(s) missing or incorrect
5(b)	Angle $ACB = 2x$	M1	May be implied by working
	$x + 2x + 2x = 180$	M1	oe eg 1 $5x = 180$ eg 2 $90 - \frac{1}{2}x = 2x$
	36	A1	
6(a)	$6x^2 - 15xy$	B2	B1 Only one correct term
6(b)	$9x^2 - 12xy + 6xy - 8y^2$	M1	oe Must have 4 terms with at least 3 correct
	$9x^2 - 12xy + 6xy - 8y^2$	A1	All 4 terms correct
	$9x^2 - 6xy - 8y^2$	A1 ft	ft From M1 A0
6(c)	3 : 2	B2 ft	ft Their (a) and their (b) with $y = 0$ substituted B1 ft Any equivalent unsimplified ratio eg $9x^2 : 6x^2$ SC1 2 : 3

Q	Answer	Mark	Comments
7(a)	$-8 \leq m + n \leq 7$	B2	B1 -8 or 7 in correct position
7(b)	$0 \leq (m + n)^2 \leq 64$	B2 ft	**If (a) is fully correct ft does not apply** B1 For 0 or 64 in correct position **If (a) is not fully correct apply ft** Can only award B2 ft if their (a) has one negative value and one positive value B1ft for one value in correct position Can award a maximum of B1 ft if in (a) both values have the same sign or one value is zero
8(a)	*C*	B1	Do not allow if more than one answer selected
8(b)	*A*	B1	Do not allow if more than one answer selected

Q	Answer	Mark	Comments
9(a)	$5t + 3 = 4wt + 8w$	M1	
	$5t - 4wt = 8w - 3$	M1	Separation of terms in t from those not in t
	$t(5 - 4w) = 8w - 3$	M1	Factorisation of terms in t
	$t = \frac{8w-3}{5-4w}$	A1 ft	oe eg $t = \frac{3-8w}{4w-5}$ Must have $t =$ **Only ft if third M1 and one other M1 gained**
9(b)	$\frac{8\times-\frac{1}{8}-3}{5-4\times-\frac{1}{8}}$	M1	Substitution of $w = -\frac{1}{8}$ in their $\frac{8w-3}{5-4w}$ Their $\frac{8w-3}{5-4w}$ must be in terms of w
	Numerator $= -4$ **or** denominator $= 5\frac{1}{2}$	A1 ft	ft Their $\frac{8w-3}{5-4w}$ This mark can only be gained for correct evaluation of their algebraic numerator **or** their algebraic denominator
	$-\frac{8}{11}$ **or** $-0.\dot{7}\dot{2}$	A1 ft	ft Their $\frac{8w-3}{5-4w}$ This mark can only be gained for correct evaluation of their algebraic numerator **and** their algebraic denominator Must be an exact value in simplest form SC2 $-0.72\ldots$ **or** -0.73 **or** a correct evaluation of their algebraic numerator or their algebraic denominator
Alt 9(b)	$5t + 3 = -\frac{4}{8}(t + 2)$	M1	oe equation
	$44t = -32$	A1	oe eg $5.5t = -4$
	$-\frac{8}{11}$ **or** $-0.\dot{7}\dot{2}$	A1 ft	ft from their $at = b$ if M1 A0 Must be an exact value in simplest form SC2 $-0.72\ldots$ **or** -0.73

Q	Answer	Mark	Comments
10	sin 28 chosen	B1	cos 62 chosen
	$\frac{7}{\sin 28}$	M1	$\frac{7}{\cos 62}$
	[14.9, 14.9104]	A1	Allow 15 if correct working for M1 seen
11	$\frac{4}{3}\pi x^3 (=) \frac{2}{3}\pi y^3$	M1	oe eg 1 $\frac{4}{3}\pi \times x^3 (=) \frac{1}{2} \times \frac{4}{3}\pi \times y^3$ eg 2 $y^3 = 2x^3$
	$(\frac{y^3}{x^3} =) \frac{\frac{4}{3}\pi}{\frac{2}{3}\pi}$ **or** $y = \sqrt[3]{2}\,x$	M1 Dep	oe eg $\frac{y^3}{x^3} = 2$
	$2^{\frac{1}{3}}$	A1	$\sqrt[3]{2}$ scores M2 A0
12	$(t + 4)(t^2 + 4t + 4t + 16)$	M1	oe Must be correct
	$t^3 + 4t^2 + 4t^2 + 16t + 4t^2 + 16t + 16t + 64$	M1	ft From their $(t + 4)(t^2 + 4t + 4t + 16)$ oe Must have at least 4 terms correct M2 $t^3 + 3t^2(4) + 3t\,(4)^2 + 4^3$ oe
	$t^3 + 12t^2 + 48t + 64$	A1	
13	$\frac{16^2 + 9^2 - 20^2}{2 \times 16 \times 9}$ (= – 0.21875)	M1	oe eg $\frac{256 + 81 - 400}{288}$ **or** $-\frac{63}{288}$ **or** –288cos$x$ = 63
	$\cos^{-1} \frac{16^2 + 9^2 - 20^2}{2 \times 16 \times 9}$	M1	oe This mark implies the first M1
	[102.6, 102.64]	A1	Allow 103 if correct working for M1 M1 seen

Q	Answer	Mark	Comments
14	x coordinate of centre = 2	B1	
	y coordinate of centre = 5	B1	
	$(x$ – their 2$)^2 + (y$ – their 5$)^2$	M1	= 25 not needed for M1
	$(x$ – their 2$)^2 + (y$ – their 5$)^2 = 25$	A1 ft	oe eg Allow 5^2 for 25 ft From their centre of circle Ignore any attempt to expand and simplify
15(a)	$3x^2 - 5$ seen	B1	
	Correct step in attempt to solve their $f(x^2) = 43$ (must be a quadratic equation) $3x^2 = 43 + 5$	M1	oe eg 1 $3x^2 - 5 - 43 = 0$ eg 2 $x^2 = \frac{43+5}{3}$
	$x^2 = 16$	A1	$(3)(x + 4)(x - 4)$
	4 **and** –4	A1 ft	ft From M1 A0 if two solutions found SC2 3.56 **and** –3.56
15(b)	(gradient for $0 \leq x \leq 4$ =) $\frac{12}{4}$ **or** 3	M1	oe
	(gradient for $4 < x \leq 8$ =) $\frac{12}{-4}$ **or** – 3	M1	oe Accept – their 3
	y = their $-3x + c$ **and** substitutes (8,0) or (4,12)	M1	y – 0 = their $-3(x - 8)$ **or** y – 12 = their $-3(x - 4)$
	$3x$ **and** $-3x + 24$ or $-3(x - 8)$ in correct places on answer lines	A2	A1 $3x$ **or** $-3x + 24$ or $-3(x - 8)$ in correct place on answer line **or** $y = 3x$ (for $0 \leq x \leq 4$) **or** $y = -3x + 24$ or $y = -3(x - 8)$ (for $4 < x \leq 8$)

Q	Answer	Mark	Comments	
16(a)	$1^3 - 21(1) + 20 = 0$ **or** $1 - 21 + 20 = 0$	B1	Must have = 0	
	$4^3 - 21(4) + 20 = 0$ **or** $64 - 84 + 20 = 0$	B1	Must have = 0	
16(b)	$1^3 - 10(1)^2 + 29(1) - 20 = 0$ **or** $1 - 10 + 29 - 20 = 0$ Divides $x^3 - 10x^2 + 29x - 20$ by $(x - 1)$ and obtains answer $x^2 - 9x + 20$	B1	Must have = 0	B2 $(x - 1)(x - 4)(x - 5)$ **and** correct expansion of one pair of brackets eg $(x - 1)(x - 4)(x - 5)$ **and** $(x^2 - 5x + 4)(x - 5)$ B1 $(x - 1)(x - 4)(x - 5)$
	$4^3 - 10(4)^2 + 29(4) - 20 = 0$ **or** $64 - 160 + 116 - 20 = 0$ Divides $x^3 - 10x^2 + 29x - 20$ by $(x - 4)$ and obtains answer $x^2 - 6x + 5$	B1	Must have = 0	
16(c)	$(x + 5)$ as 3rd factor of numerator	B1	Implied by final answer $\frac{x+5}{ax+b}$	
	$(x - 5)$ as 3rd factor of denominator	B1	Implied by final answer $\frac{cx+d}{x-5}$	
	$\frac{\text{their } x+5}{\text{their } x-5}$	B1 ft	Do **not** award if further work	
17	CE or EB = $2x$ **or** DF = x **or** FC = $3x$ **or** area ABCD = $16x^2$	B1	May be on diagram or implied in working	
	Area ABE = $\frac{1}{2} \times$ their $2x \times 4x$ $(= 4x^2)$ **and** area CFE = $\frac{1}{2} \times$ their $2x \times$ their $3x$ $(= 3x^2)$ **and** area ADF = $\frac{1}{2} \times$ their $x \times 4x$ $(= 2x^2)$	M2	Attempt at all three triangle areas ABE and CFE and ADF M1 Attempt at any one triangle area ABE or CFE or ADF All areas must be in terms of x	
	$4x \times 4x$ – their $4x^2$ – their $3x^2$ – their $2x^2$ $(= 7x^2)$	M1 Dep	Dep on at least M1 All areas must be in terms of x^2	
	7	A1		

Q	Answer	Mark	Comments
18	$a = 3$ **and** $b = -10$	B3	B2 $a = 3$ **or** $b = -10$ B1 $x^2 - 5x - 5x + 25$ oe
19	$(4 - x)^2 = 4x + 5$	M1	
	$16 - 4x - 4x + x^2 = 4x + 5$	M1 Dep	Allow one error but must be a quadratic in x
	$x^2 - 12x + 11$ (= 0)	A1	oe Must be 3 terms
	$(x - 11)(x - 1)$ (= 0)	M1	$\frac{--12 \pm \sqrt{(-12)^2 - 4(1)(11)}}{2}$ **or** $(x - 6)^2 - 36 + 11 = 0$ oe
	$x = 11$ and $x = 1$	A1 ft	Must have M3 to ft $x = 11$ and $y = -7$ **or** $x = 1$ and $y = 3$
	$x = 11$ and $y = -7$ **and** $x = 1$ and $y = 3$	A1	
Alt 19	$y^2 = 4(4 - y) + 5$	M1	
	$y^2 = 16 - 4y + 5$	M1 Dep	Allow one error but must be a quadratic in y
	$y^2 + 4y - 21$ (= 0)	A1	oe Must be 3 terms
	$(y + 7)(y - 3)$ (= 0)	M1	$\frac{-4 \pm \sqrt{4^2 - 4(1)(-21)}}{2}$ **or** $(y + 2)^2 - 4 - 21 = 0$ oe
	$y = -7$ and $y = 3$	A1 ft	Must have M3 to ft $x = 11$ and $y = -7$ **or** $x = 1$ and $y = 3$
	$x = 11$ and $y = -7$ **and** $x = 1$ and $y = 3$	A1	

Q	Answer	Mark	Comments
20	$150 - 6x^2$	B1	
	their $150 - 6x^2 > 0$ **or** their $150 - 6x^2 = 0$	M1	their $150 - 6x^2$ must be in terms of x Must be > 0 or $= 0$
	$\frac{150}{6} > x^2$ or $(6)(5 - x)(5 + x)$ (> 0) **or** $\frac{150}{6} = x^2$ or $(6)(5 - x)(5 + x)$ $(= 0)$	M1 Dep	ft Their inequality **only if a quadratic** either simplified to $k > x^2$ or factorised correctly **or** ft Their equation **only if a quadratic** either simplified to $k = x^2$ or factorised correctly
	$-5 < x < 5$	A1	Allow $x > -5$ and $x < 5$ (must have both inequalities as well as the 'and')
21	Fully correct method to eliminate a letter from *OB* and *AB* $2(2x) = 11x - 7$	M1	oe eg 1 $2y = 11(\frac{y}{2}) - 7$ eg 2 $2y - 4x = 0$ $2y - 11x = -7$ **and** $7x = 7$
	Coordinates of $B = (1, 2)$	A1	Implied by $x = 1$ and $y = 2$
	Fully correct method to eliminate a letter from *OA* and *AB* $2y = 11(-3y) - 7$	M1	oe eg 1 $x + 3(\frac{11x - 7}{2}) = 0$ eg 2 $2x + 6y = 0$ $33x - 6y = 21$ **and** $35x = 21$
	Coordinates of $A = (0.6, -0.2)$	A1	oe Implied by $x = 0.6$ and $y = -0.2$
	OB^2 = their 1^2 + their 2^2 **or** AB^2 = (their 1 – their 0.6)^2 + (their 2 – their –0.2)^2	M1	oe eg correct attempt at *OB* **or** *AB* ft Their *B* and / or their *A*
	$OB = \sqrt{5}$ **and** $AB = \sqrt{5}$	A1	oe eg $OB^2 = 5$ **and** $AB^2 = 5$

Q	Answer	Mark	Comments
22	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-4\\3\end{pmatrix}$	M1	
	$\begin{pmatrix}x\\-y\end{pmatrix}=\begin{pmatrix}-4\\3\end{pmatrix}$ **or** $\begin{pmatrix}x\\y\end{pmatrix}=\begin{pmatrix}-4\\-3\end{pmatrix}$ **or** Q (–4, –3)	A1	oe
	$\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}\begin{pmatrix}X\\Y\end{pmatrix}$ = their $\begin{pmatrix}-4\\-3\end{pmatrix}$	M1 Dep	
	$\begin{pmatrix}-Y\\-X\end{pmatrix}=\begin{pmatrix}-4\\-3\end{pmatrix}$	A1 ft	oe ft Their $\begin{pmatrix}-4\\-3\end{pmatrix}$ if M2 gained
	(3, 4)	A1 ft	ft Their $\begin{pmatrix}-4\\-3\end{pmatrix}$ if M2 gained SC4 $\begin{pmatrix}3\\4\end{pmatrix}$
Alt 1 **22**	$\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$	M1	This order only
	$\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$	A1	
	Their $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$	M1 Dep	
	$\begin{pmatrix}-y\\x\end{pmatrix}=\begin{pmatrix}-4\\3\end{pmatrix}$	A1 ft	oe ft Their $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ if M2 gained
	(3, 4)	A1 ft	ft Their $\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$ if M2 gained SC3 (–3, –4) SC4 $\begin{pmatrix}3\\4\end{pmatrix}$

Q	Answer	Mark	Comments
Alt 2 **22**	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}$	M1	
	$\begin{pmatrix} -y \\ -x \end{pmatrix}$	A1	
	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ their $\begin{pmatrix} -y \\ -x \end{pmatrix}$	M1 Dep	
	$\begin{pmatrix} -y \\ x \end{pmatrix} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$	A1 ft	oe ft Their $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ if M2 gained
	(3, 4)	A1 ft	ft Their $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ if M2 gained SC4 $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$
Alt 3 **22**	Attempt to reflect (–4, 3) in the x-axis	M1	
	(–4, –3)	A1	
	Attempt to reflect their (–4, –3) in the line $y = -x$	M1 Dep	
	(3, 4)	A2 ft	ft Their (–4, –3) if M2 gained SC4 $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$

Q	Answer	Mark	Comments
23	Trials values either side of $x = 0$ $x = -1$ $\frac{dy}{dx} = 9$ **and** $x = 1$ $\frac{dy}{dx} = -1$	M1	oe Allow statements that $\frac{dy}{dx}$ is positive/negative but any evaluations seen must be correct
	Maximum $(0, \frac{4}{3})$	A1	Can only be awarded with correct method seen
	Trials values either side of $x = 2$ $x = 1$ $\frac{dy}{dx} = -1$ (may have been seen earlier) $x = 3$ $\frac{dy}{dx} = -3$	M1	oe Allow statements that $\frac{dy}{dx}$ is negative but any evaluations seen must be correct
	(Point of) inflection (2, 0)	A1	Can only be awarded with correct method seen
Alt 23	$\frac{d^2y}{dx^2} = -3x^2 + 8x - 4$ **and** substitutes $x = 0$ **and** $\frac{d^2y}{dx^2} = -4$	M1	Second derivative must be correct Allow statement that $\frac{d^2y}{dx^2}$ is negative but if evaluation seen it must be correct
	Maximum $(0, \frac{4}{3})$	A1	Can only be awarded with correct method seen
	Trials values either side of $x = 2$ $x = 1$ $\frac{dy}{dx} = -1$ $x = 3$ $\frac{dy}{dx} = -3$	M1	oe eg uses second and third derivatives Allow statements that $\frac{dy}{dx}$ is negative but any evaluations seen must be correct
	(Point of) inflection (2, 0)	A1	Can only be awarded with correct method seen